

Decompression Theory

Jaap Voets

Chapter 1

Decompression theory

This document accompanies the perl module Deco.pm. It will give you a basic background on the Haldanian model used in the module. Many thanks go to the authors of www.deepocean.net who created the text from which I borrowed heavily.

1.1 History

In modern diving, tables and schedules are common for estimating no-decompression limits, decompression profiles and saturation levels. Use of a diving computer during the dive is most common nowadays. Tables and computers are based on decompression theory, which describes inert gas uptake and saturation of bodily tissue when breathing compressed air (or other gas mixtures). The development of this theory was started in 1908 by John Scott Haldane c.s. Haldane, an English physiologist, described the Nitrogen saturation process by using a body model which comprises several hypothetical tissue 'compartments'.

A compartment can be characterized by a variable called 'half-time', which is a measure for inert gas uptake. Theory was further developed during the years '50 and '60 by U.S. Navy. The concept of 'M-values' was developed by Robert D. Workman of the U.S. Navy Experimental Diving Unit (NEDU). In the early '70s Schreiner applied the theory to changing pressure (ascending/descending). Recently Bhlmann improved the theory and developed ZH-12L and ZH-16L model, which are quite popular in current diving computers. At this moment a lot is still unknown about the exact processes which take place during saturation and decompression. Most of the

theory presented has been found empirically, i.e. by performing tests on human subjects in decompression chambers and from decompression accident statistics. Recently, a more physical approach resulted in bubble theories. These theories physically describe what is happening during decompression.

1.2 Inert gas saturation and supersaturation

When you breathe a breathing gas that contains an inert gas (gasses which do not take part in the oxidative metabolism and are not 'used' by the body) like Nitrogen (N_2) or Helium (He), this gas is dissolved in the blood by means of gas exchange in the lungs. Blood takes the dissolved gas to the rest of the bodily tissue. Tissue takes up dissolved gas from the blood. Gas keeps on dissolving in blood and tissue until the partial pressure of the dissolved gas is equal to the partial pressure of the gas breathed in, throughout the entire body. This is called saturation. Rates of saturation vary with different parts of the body. The nervous system and spine get saturated very fast (fast tissues), whereas fat and bones saturate very slowly (slow tissues).

When staying at sea level for a long time, like most of us do, and breathing air, again like most of us do, the entire body is saturated with Nitrogen, which makes up the air for 78%. Since at sea level air pressure is roughly 1 bar (we can neglect barometric air pressure variations, which are expressed in milli-bar), the partial pressure of the dissolved Nitrogen throughout the entire body is $1 \text{ bar} * 78\% = 0.78 \text{ bar}$ (actually it is a bit less, as we will see later, but for the moment this will do). If a diver dives to 20 meter, he breathes air at 3 bar. Partial Nitrogen pressure in the air he breathes is $3 \text{ bar} * 78\% = 2.34 \text{ bar}$. If the diver sits down and wait for quite a long time, the diver's body gets saturated with Nitrogen at a partial pressure of 2.34 bar (the fastest tissues saturate in 25 minutes, the slowest take two and a half day to saturate). So far, so good (at least if our diver has enough air supply).

If the diver goes back to the surface, however, he arrives with his body saturated with Nitrogen at a partial 2.34 bar pressure, whereas the air he breathes at the surface has a partial Nitrogen pressure of 0.78 bar. The body is supersaturated. Dissolved Nitrogen in the tissue and blood will go back to the free gas phase, in order to equalize the pressures. The Nitrogen forms micro bubbles, which are transported by the blood and removed from the body by the respiratory system. However, if too much Nitrogen goes back to

the free phase, micro bubbles grow and form bubbles that may block veins and arteries. The diver gets bent and will develop decompression sickness (DCS) symptoms.

A certain amount of supersaturation is allowed, without getting bent (at least with low risk of getting bent). In fact, supersaturation (a pressure gradient) is needed in order to decompress (get rid of the excess Nitrogen). The amount of allowed supersaturation is different for various types of tissue. This is the reason the body is divided in hypothetical tissue 'compartments' in most decompression models. Each compartment is characterized by its half-time. This is the period the tissue takes a partial inert gas pressure which is half way between the partial pressure before and after a pressure change of the environment. Haldane suggested two compartments, recent theories (like ZH-16L) use up to 16 compartments. Decompression theory deals with two items:

- Modeling the inert gas absorption in the bodily tissue
- Estimate limits of supersaturation for each tissue, beyond which decompression sickness (DCS) symptoms develop

If these items are known, one can fill in tables, estimate no-decompression times, calculate decompression profiles, plan dives, etc.

1.3 Calculating inert gas absorption by the tissue

In this section we will derive the equations which describe inert gas uptake by bodily tissue. If you get frightened by a bit of mathematics, please skip the derivation, but have a look at the end result in the high lighted boxes (equation (1.8) and (1.13)). The rate a particular tissue (compartment) takes up inert gas (i.e. the rate of change in partial pressure of that gas in the tissue) is proportional to the partial pressure difference between the gas in the lungs and the dissolved gas in the tissue. We can express this mathematically by:

$$\frac{d}{dt}P_t(t) = k[P_{alv}(t) - P_t(t)] \quad (1.1)$$

where

- $P_t(t)$ Partial pressure of the gas in the particular tissue (*bar*)

- $P_{alv}(t)$ Partial pressure of the gas in the breathing mix. To be precise: Gas exchange takes place in the lungs (alveoli). Hence, we have to consider the gas alveolar partial pressure. This pressure may be changing with time, if the diver changes depth (*bar*)
- k A constant depending on the type of tissue (min^{-1})
- t Time (*min*)

This is a differential equation which is quite familiar in physics and apply to many processes like diffusion and heat transfer. Solving this equation requires following steps:

1. Write the equation to the familiar form (a.k.a. the inhomogenous differential equation):

$$\frac{d}{dt}P(t) + kP(t) = kP_{alv}(t) \quad (1.2)$$

2. Solve the homogenous equation (1.3) by trying $P_{th}(t) = C_0e^{-\lambda t}$ and solve for λ .

$$\frac{d}{dt}P_{th}(t) + kP_{th}(t) = 0 \quad (1.3)$$

The 'h' in P_{th} denotes that we are dealing with the homogenous equation. Substituting the solution in (1.3) results in $\lambda = k$. So the homogenous solution of the equation (1.3) is:

$$P_{th}(t) = C_0e^{-kt} \quad (1.4)$$

3. Find the particular solution for the inhomogenous equation (1.2) and solve constants using boundary conditions. In order to solve this equation, we have to know more about the partial gas pressure $P_{alv}(t)$. Two useful situations described in literature are a situation in which $P_{alv}(t)$ is constant (corresponding to remaining at a certain depth) and a situation in which $P_{alv}(t)$ varies linearly with time (corresponding to ascending/descending with constant speed). We will have a look at both situations.

1.3.1 constant ambient pressure

We look at the situation in which the alveolar partial pressure of the gas remains constant: $P_{alv}(t) = P_{alv0}$. This corresponds to a diving situation in which the diver remains at a certain depth. Equation (1.2) becomes:

$$\frac{d}{dt}P_t(t) + kP_t(t) = kP_{alv0} \quad (1.5)$$

We 'try' the solution:

$$P_t(t) = C_0 e^{-kt} + C_1 \quad (1.6)$$

If we substitute solution (1.6) in equation (1.5) the e's cancel out and we are left with $C_1 = P_{alv0}$. Now we have to think of a boundary condition, in order to find C_0 . We assume some partial pressure in the tissue $P_t(0) = P_{t0}$ at $t = 0$. If we substitute this into equation (1.6) we find that $C_0 = [P_{t0} - P_{alv0}]$. So we are left with the following equation for the partial pressure in a specific type of tissue (characterized by the constant k):

$$P_t(t) = P_{alv0} + [P_{t0} - P_{alv0}]e^{-kt} \quad (1.7)$$

where

- $P_t(t)$ Partial pressure of the gas in the tissue (*bar*)
- P_{t0} Initial partial pressure of the gas in the tissue at $t=0$ (*bar*)
- P_{alv0} Constant partial pressure of the gas in the breathing mix in the alveoli (*bar*)
- k A constant depending on the type of tissue (min^{-1})
- t Time (*min*)

This equation is known in literature as the **Haldane equation**. We can rewrite it a bit so that it corresponds to a form which is familiar in decompression theory literature:

$$P_t(t) = P_{t0} + [P_{alv0} - P_{t0}][1 - e^{-kt}] \quad (1.8)$$

1.3.2 linearly varying ambient pressure

Very few divers plunge into the deep and remain at a certain depth for a long time. For that reason we will look at the situation in which the diver ascends or descends with constant speed. This means the partial pressure of the gas he breathes varies linearly with time. Going back to equation (1.2) this means P_{alv} can be written as $P_{alv} = P_{alv0} + Rt$ P_{alv0} is the initial partial pressure of the gas in the breathing mixture at $t=0$, and R is the change rate (in bar/minute) of the partial pressure of this gas in the alveoli. Note: R is positive for descending (pressure increase) and negative for ascending (pressure decrease). Substituting this in (1.2) gives us:

$$\frac{d}{dt}P_t(t) + kP_t(t) = kP_{alv0} + kRt \quad (1.9)$$

We 'try' the solution:

$$P_t(t) = C_0e^{-kt} + C_1t + C_2 \quad (1.10)$$

Substituting solution (1.10) in equation (1.9) leaves us with:

$$[kC_1 - kR]t + [C_1 + kC_2 - kP_{alv0}] = 0 \quad (1.11)$$

To find a solution for C_1 and C_2 that hold for every t we have to make both parts between the square brackets in (1.11) equal to 0. This results in $C_1 = R$ and $C_2 = P_{alv0} - \frac{R}{k}$. In this way we find:

$$P_t(t) = C_0e^{-kt} + Rt + P_{alv0} - \frac{R}{k} \quad (1.12)$$

Again we use as boundary condition $P_t(0) = P_{t0}$ at $t=0$ in order to find C_0 . Substituting this in (1.12) we find $C_0 = P_{t0} - P_{alv0} + R/k$. So for the ultimate solution we find:

$$P_t(t) = P_{alv0} + R[t + \frac{R}{k}] - [P_{alv0} - P_{t0} - \frac{R}{k}]e^{-kt} \quad (1.13)$$

where

- $P_t(t)$ Partial pressure of the gas in the tissue (bar)

- P_{t0} Initial partial pressure of the gas in the tissue at $t=0$ (bar)
- P_{alv0} Initial (alveolar) partial pressure of the gas in the breathing mix at $t=0$ (bar)
- k A constant depending on the type of tissue
- R Rate of change of the partial inert gas pressure in the breathing mix in the alveoli (bar/min) $R=Q \text{ R}_{amb}$, in which Q is the fraction of the inert gas and R_{amb} is the rate of change of the ambient pressure.
- t Time (min)

This solution was first proposed by Schreiner and hence known as the **Schreiner equation**. If we set the rate of change R to 0 (remaining at constant depth), the equation transforms in the Haldane equation (1.8). The Schreiner equation is excellent for application in a simulation as used in diving computers. With the same frequency of measuring of the environment pressure and performing the calculation, applying the Schreiner equation gives a more precise approximation of the actual pressure profile in the bodily tissue than the Haldane equation.

1.4 Half times

So we see an exponential behavior. When we look at the first situation (constant depth) we have a tissue with in initial partial pressure P_{t0} . Eventually the partial pressure of gas in the tissue will reach the partial pressure of the gas in the breathing mixture P_{alv0} . We can calculate how long it takes for the partial pressure to get half way in between, i.e. $e^{-k\tau} = \frac{1}{2}$. The variable τ (tau) is called the 'half-time' and is usually used for characterizing tissue (compartments). Rewriting: $-k\tau = \ln\left(\frac{1}{2}\right) = -\ln(2)$. So the relation between k and the half-time τ is:

$$\tau = \frac{\ln(2)}{k} \quad (1.14)$$

and

$$k = \frac{\ln(2)}{\tau} \quad (1.15)$$

1.5 The alveolar partial pressure

So far we did not worry about the values of P_{alv} . We will have a closer look at this alveolar partial pressure of the inert gas and how it is related to the ambient pressure. The pressure of the air (or gas mixture) the diver breathes is equal to the ambient pressure P_{amb} surrounding the diver. The ambient pressure depends on the water depth and the atmospheric pressure at the water surface. To be precise: it is equal to the atmospheric pressure (1 bar at sea level) increased with 1 bar for every ten meters depth. The partial pressure of the inert gas in the alveoli depends on several factors:

- The partial pressure (fraction Q) of the inert gas in the air or gas mixture breathed in
- The water vapor pressure. The dry air breathed in is humidified completely by the upper airways (nose, larynx, trachea). Water vapor dilutes the breathing gas. A constant vapor pressure at 37 degrees Celsius of 0.0627 bar (47 mm Hg) has to be subtracted from the ambient pressure
- Oxygen O_2 is removed from the breathing gas by respiratory gas exchange in the lungs
- Carbon Dioxide CO_2 is added to the breathing gas by gas exchange in the lungs. Since the partial pressure of CO_2 in dry air (and in common breathing mixtures) is negligible, the partial pressure of the CO_2 in the lungs will be equal to the arterial partial pressure. This pressure is 0.0534 bar (40 mm Hg).

The process of Oxygen consumption and Carbon Dioxide production is characterized by the respiratory quotient RQ , the volume ratio of Carbon Dioxide production to the Oxygen consumption. Under normal steady state conditions the lungs take up about 250 ml of Oxygen, while producing about 200 ml of Carbon Dioxide per minute, resulting in an RQ value of about $200/250=0.8$. Depending on physical exertion and nutrition RQ values range from 0.7 to 1.0. Schreiner uses $RQ=0.8$, US Navy uses $RQ=0.9$ and Bhlmann uses 1.0.

The alveolar ventilation equation gives us the partial pressure of the inert gas with respect to the ambient pressure:

$$P_{alv} = [P_{amb} - P_{H_2O} - P_{CO_2} + \Delta P_{O_2}]Q \quad (1.16)$$

or after rewriting

$$P_{alv} = [P_{amb} - P_{H_2O} + \frac{1 - RQ}{RQ} P_{CO_2}]Q \quad (1.17)$$

where

- P_{alv} Partial pressure of the gas in the alveoli (bar)
- P_{amb} Ambient pressure, i.e. the pressure of the breathing gas(bar)
- P_{H_2O} Water vapor pressure, at 37 degrees Celsius 0.0627 bar (47 mm Hg)
- P_{CO_2} Carbon Dioxide pressure, we can use 0.0534 bar (40 mm Hg)
- ΔP_{O_2} Decrease in partial Oxygen pressure due to gas exchange in the lungs
- RQ Respiratory quotient: ratio of Carbon Dioxide production to Oxygen consumption
- Q Fraction of inert gas in the breathing gas. For example N2 fraction in dry air is 0.78

The Schreiner RQ value is the most conservative of the three RQ values. Under equal circumstances using the Schreiner value results in the highest calculated partial alveolar pressure and hence the highest partial pressure in the tissue compartments. This leads to shorter no decompression times and hence to less risk for DCS.

1.6 Example

We will have a look at our diver who plunges to 30 m and stays there for a while. The diver breathes compressed air and did not dive for quite a while before this dive. So at the start of the dive, all his tissue is saturated with Nitrogen at a level that corresponds to sea level. We neglect the period of descending. In particular, we will look at two types of tissue in his body with a half time of 4 minutes (the fastest tissue) resp. 30 minutes (medium fast tissue). The ambient pressure at 30 meters is 4 bar. Equation (1.17) gives us a partial alveolar N2 pressure of 3.08 bar at 30 meters and 0.736 bar at sea level, using the RQ=0.9 value of the US Navy. Substituting these values in equation (6) result in (14), predicting the partial pressure in the tissues. This pressure is shown in figure 1: