# USING THE WAVEGUIDE MESH IN MODELLING 3D RESONATORS

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## ABSTRACT

Most of the results found by several researchers, during these years, in physical modelling of two dimensional (2D) resonators by means of waveguide meshes, extend without too much difficulty to the three dimensional (3D) case. Important parameters such as the dispersion error, the spatial bandwidth, and the sampling efficiency, which characterize the behavior and the performance of a waveguide mesh, can be reformulated in the 3D case, giving the possibility to design mesh geometries supported by a consistent theory.

A comparison between different geometries can be carried out in a theoretical context. Here, we emphasize the use of the waveguide meshes as efficient tools for the analysis of resonances in 3D resonators of various shapes. For this purpose, several mesh geometries have been implemented into an application running on a PC, provided with a graphical interface that allows an easy input of the parameters and a simple observation of the consequent system evolution and the output data. This application is especially expected to give information on the modes resonating in generic 3D shapes, where a theoretical prediction of the modal frequencies is hard to do.

## 1. INTRODUCTION

Multidimensional resonators can be found in all musical instruments and in almost all listening contexts. Hence, the aim to model them and to simulate their behavior by means of stable, versatile and easy-to-handle numerical methods is strongly felt. Recently, Waveguide Meshes (WM) have been proposed and carefully studied by several researches, as structures especially devoted to model wave propagation along an ideal, multidimensional medium [1, 2].

Waveguide meshes are built by interconnecting Digital Waveguides [3] according to several topologies, and provide computational structures equivalent to a Finite Difference Scheme (FDS) [4, 5]. Hence, most of the FDS theory can be recovered from the theoretical analysis of the WM: in particular, it can be shown that these structures introduce numerical artifacts that can be interpreted in terms of dispersion error. This means that, even in the simulation of a non-dispersive medium, different spatial frequency components travel at different speeds, and this speed is direction- and frequency-dependent. The dispersion functions vary with the topology of the mesh, but in any case they cause a misalignment of the resonant modes from their theoretical positions [6].

At the same time, the waveguide approach results to be more intuitive and meaningful in several questions related with these models, such as the design of the boundaries and the choice of the mesh topology. Using this approach, it has been recently understood that the dispersion error, introduced by discretizing in time and space the propagation medium, can be reduced in certain topologies using interpolation, and off line or online warping techniques [7, 8]. These techniques increase the cost of the resulting model less than a reduction of the dispersion obtained by mere oversizing of the mesh.

Definitely, research in WMs is leading to interesting models of resonators, even if much still needs to be done. For instance, there is lack of simulations producing natural sounds, although the WM should couple without too much difficulty with realistic damping elements and other resonating structures.

With this goal in mind, we have started to implement various WM geometries into an application running on a PC. In particular, the application embeds two new 3D geometries still unseen in the literature, for which a brief theoretical discussion is given in the next section. The application accepts parameters as mesh geometry, junctions' density, shape of the resonator, type of excitation and positions of the listening points. Together with them, more complex parameters such as the online warping factor, here obtained by cascading the digital waveguides with properly tuned all-pass filters, allow to control dispersion and/or to shift dynamically the modes. The application includes a visual interface that makes the software a useful tool for the analysis of ideal resonators and mesh structures, and a versatile building block for future developments.

## 2. 3D SCHEMES

In the 2D case, the geometries that can be chosen are not so many. Among the available ones, the literature suggests to select the triangular or the de-interpolated WM, which have the most regular behavior of the dispersion, or the square geometry, which is a good trade-off between computational cost and simplicity of implementation of its 4-port junctions [7, 8].

Once the third dimension is added, the collection of available geometries becomes richer: it includes the orthogonal WM, already exploited in simulation of room acoustics [9], the tetrahedral WM [10], and other, sometimes complex, topologies. In order to make a good choice in this panorama, it should be proved, as done in the 2D case [11], that

- geometries in which the junctions have more than one orientation (i.e.: the tetrahedral) in general do not translate into efficient models, due to the gaps present in the underlying sampling scheme;
- meshes where the signals, traveling from one (input) junction to another (output) junction, run through a number of waveguides which is always even or always odd, define input-output transfer functions in the variable $z^{-2}$. Hence,

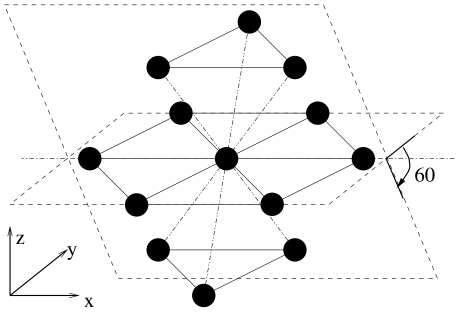


Figure 1: *The 3DTWM.*

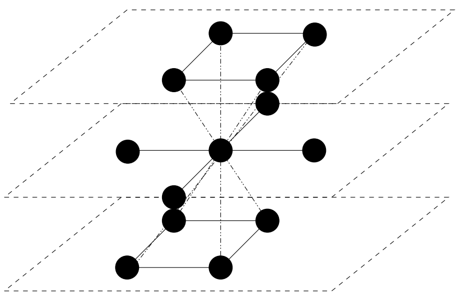

Figure 2: *Superposition of interlaced square WMs.*

their frequency response mirrors at half the Nyquist frequency;

- geometries resulting in particular distributions of the junctions in the 3D space, map themselves into non-orthogonal, often efficient sampling schemes;
- WMs exhibiting a uniform characteristic of the dispersion error, i.e. a good degree of symmetry of the dispersion function around the origin, allow a reduction of this error by means of frequency warping techniques.

In particular, we have chosen two geometries satisfying these assumptions. The former, which we are calling 3D triangular WM (3DTWM), is presented in figure 1: it is obtained by superposing four triangular WMs, through rotations of $60^\circ$ of one triangular WM along the three axes parallel to the waveguide directions (in figure 1, one plan obtained by the rotation along one of these axes is depicted). The latter is defined by the superposition along the third dimension of planes containing square WMs, in such a way that each couple of adjacent planes is shifted of half the waveguide length in both waveguide direction (see figure 2), so creating interlaced adjacent square WMs, separated by a distance tuned to equate the length of all the waveguides composing the 3D structure.

Both the geometries are made of junctions, each one having only one orientation. Both schemes allow walks of the signal through even or odd numbers of waveguides. Both of them lie on efficient, non-orthogonal sampling schemes. Finally, both of them — especially the 3DTWM — have a very uniform dispersion characteristic, as shown in figures 3 and 4, where projections of the dispersion function over the plan $(x, z)$ of the normalized frequency domain are plotted.

In conclusion, the two geometries can be adopted in practice as efficient structures to model 3D resonators.

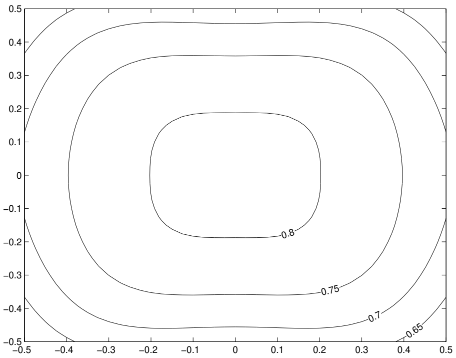


Figure 3: *Projection of the dispersion function over the plan $(x, z)$ of the normalized frequency domain in the 3DTWM.*

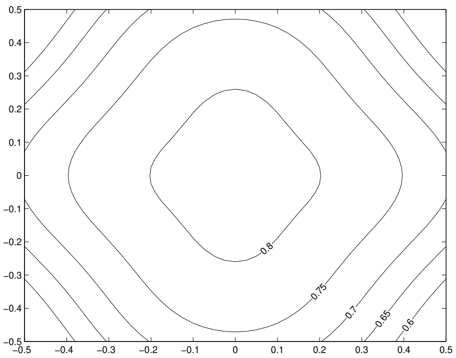


Figure 4: *Projection of the dispersion function over the plan $(x, z)$ of the normalized frequency domain in the superposition of interlaced square WMs.*

## 3. IMPLEMENTATION

Several geometries have been implemented in an application — still unnamed — written in C++ language. The scattering junctions, together with their waveguide neighbors, have been designed as objects, in such a way that a mesh is generated using a constructor, which builds up the structure according to the dimensions and the shape of the resonator, and to the mesh geometry. For this reason, the resonator turns out to be a composition of atomic volumes, where each volume is filled with one object. In this way it is possible to compose resonators with given shape and dimensions, using the desired geometry of the WM (figure 5).

Further parameters may be selected. In particular, a coefficient of attenuation, which controls the decay of the signals, can be set. Meanwhile, a warping coefficient, controlling the parameter of first-order all pass filters embedded in the meshes, can be tuned according with the characteristics of the resonator. Finally, one or more input signals can be injected in any junction; in particular, when a 2D circular resonator of radius $R$ and tension $T$ is modeled, the input function of displacement $d$ can be set in the form

$$d(r) = \begin{cases} \frac{F}{2\pi T}\left[\ln\frac{R}{s} + \frac{1}{2s^2}(s^2 - r^2)\right] & , \quad 0 \le r < s \\ \frac{F}{2\pi T}\ln\frac{R}{s} & , \quad s \le r < R \end{cases} ,$$

corresponding to the initial condition for the displacement in a

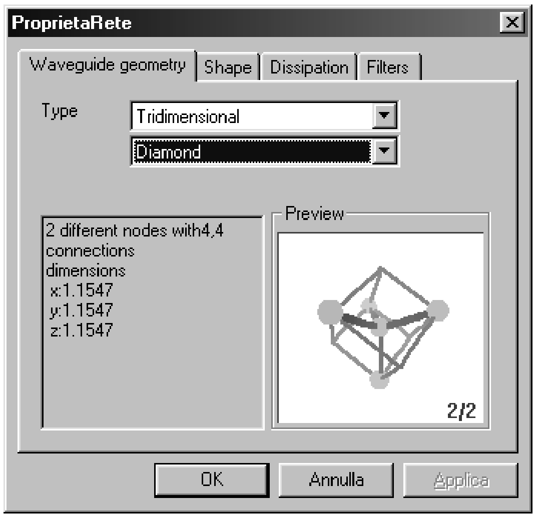


Figure 5: *The input window in the application.*

point $r$ units far from the center, when a uniform force $F$ is applied to an area of radius $s$ centered over the membrane [12].

The system evolution can be monitored step by step by direct observation of the signal in correspondence of the junctions, in several ways. In particular, the signal traveling inside a 3D resonator can be observed by the colors assumed by the junctions, or by visualization of “slices” of the resonator, each one being one of the superimposed plans composing the solid shape. Otherwise, the application can be made running indefinitely.

Similarly, one or more output junctions can be chosen in the mesh. The output signals or, equivalently, their spectra, can be monitored during the system evolution, and saved as raw files that can be easily transformed into new data structures for feeding more complex analysis tools as Matlab$^{\text{TM}}$.

The application, still at its first steps, has already allowed to do some interesting observations about the evolution of the wave signals traveling along the mesh. In particular, it has confirmed the high degree of redundancy present in the signal, when low-efficiency geometries are selected. In the 3D case, this implies a large amount of operations that can be avoided adopting more efficient schemes.

Soon, a functionality for the creation of general parametric shapes will be embedded in the application. Hybrid resonators will be obtained by generating mesh structures according with the equations that express possible transitions between shapes, whose resonance modes are well known in physical acoustics (like, for example, a cube or a sphere). By this improvement in the application, we expect to study the sounds coming out from hybrid 3D resonators, inspired in this by studies on the perception of spatial relationships conducted in the field of visual perception and image synthesis [13] and, at a starting level, in the field of psychophysical acoustics [14].

By exploiting the object nature of the software, further structures could be embedded in the code, to implement other typical features which characterize the resonators, like wall absorption or wave diffusion, and to couple two or more different resonators. This goal needs further theoretical investigations about the best way to integrate these features into the waveguide paradigm, and could be hopefully achieved by sharing the code between all the research communities interested in the analysis and the application of the WM.

## 4. SUMMARY

Waveguide meshes are coming to a point of maturity. The underlying theory allows now to determine the best geometry in the 2D case, respect to requirements of precision and cost of the model. The same theory should extend with not much effort to the third dimension, so helping in determining the most efficient and useful geometries devoted to study the resonances produced by a 3D resonator.

The implementation of several geometries into an application running on a common PC seems to confirm this assumption. Moreover, the same application shows interesting aspects in the system evolution of the waveguide models of 3D resonators, such that it can be considered as a first, general building block in the realization of a more complete simulator of waveguide models.

The object structure of the code makes this application a versatile software, where further features will be hopefully added, once the theory of waveguide meshes will be developed toward the solution of aspects related with the modeling of real resonators, holding the condition that a community of researchers will be interested in the growth of the applications for waveguide mesh simulation.

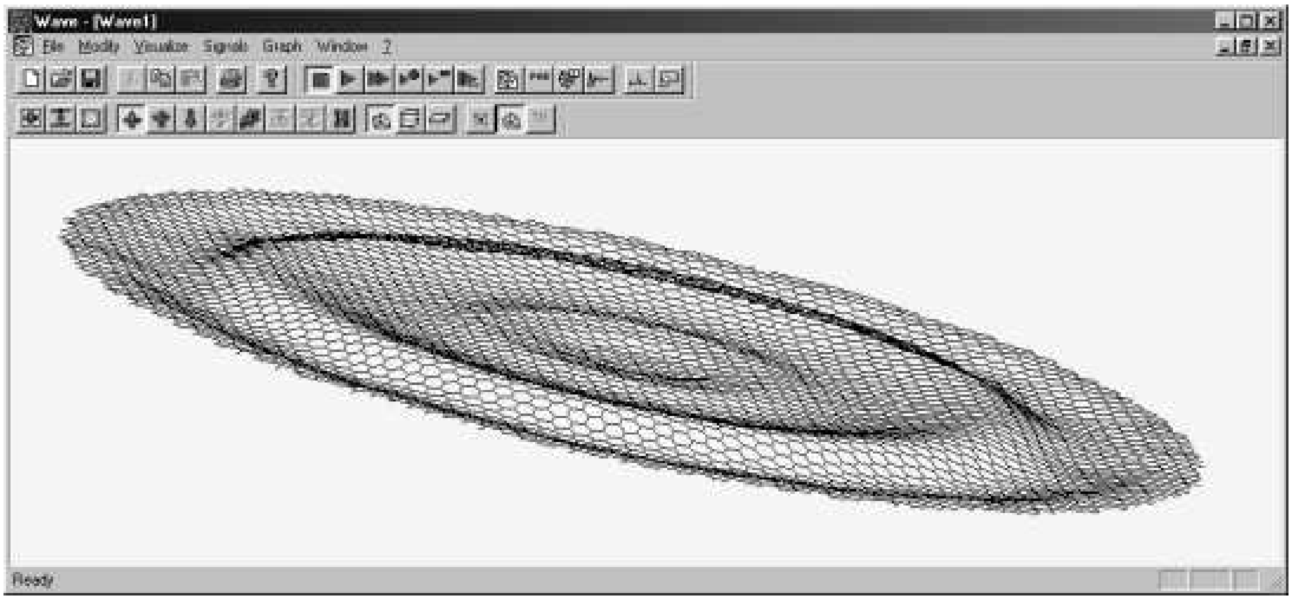


Figure 6: *The application. The visual interface allows to monitor the system evolution.*

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