A Note on "Impossible" Paper Folding

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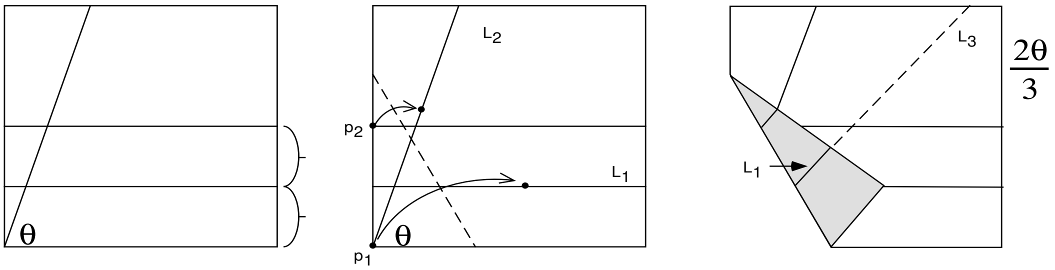
In [1] the authors draw the line between geometric constructions made with a compass and straight edge and those made by folding a sheet of paper. They wrap things up nicely with the following:

Corollary. *Every thing which is constructible with origami is constructible with a compass and straight edge, but the converse is not true.*

Amazingly enough, and without contradicting the above corollary, we present a paper folding method of trisecting an angle which was developed by Hisashi Abe in the 1970's.



The proof of this method is a simple exercise in similar triangles. Note that although we assumed θ to be acute, this method can be easily extended to obtuse angles.

The reason that this method does not contradict the above Corollary is because in [1] the authors define *origami* (i.e., paper folding) by five "origami axioms". These axioms were selected for their basic simplicity, but don't encompass all possible folding operations. Indeed, step 2 of the above trisection method uses a fold not covered in their axioms. To include this method in [1]'s modeling of paper folding, we would have to include another axiom:

vi) Given distinct points p_1 and p_2 and distinct nonparallel lines L_1 and L_2, there exists a unique fold which places p_1 onto L_1 and p_2 onto L_2.

In trying to make an axiomatic model of geometric constructions via paper folding one might want to first look at what origamists have been able to do. In the origami literature there are methods for angle trisection ([5], [6]), doubling cubes ([6], but also see [7]), and folding regular heptagons ([8]), all done using simple folding operations. However, determining the limits of origami constructions is an open problem.

References

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[3] E. Frigerio, H. Huzita, A Possible Example of System Expansion in Origami Geometry, in *Proceedings of the First International Meeting of Origami Science and Technology*, ed. H. Huzita (1989), 53-69.

[4] H. Huzita, Axiomatic Development of Origami Geometry, in *Proceedings of the First International Meeting of Origami Science and Technology*, ed. H. Huzita (1989), 143-158.

[5] H. Huzita, The Trisection of a Given Angle Solved by the Geometry of Origami, in *Proceedings of the First International Meeting of Origami Science and Technology*, ed. H. Huzita (1989), 195-214.

[6] K. Kasahara, *Origami: El Mundo Nuevo* (in Japanese), Sanrio Co., Japan (1989), 262-263.

[7] P. Messner, Problem 1054 in *Crux Mathematicorum*, **12**, No. 10 (1986).

[8] B. Scimemi, Draw of a Regular Heptagon by the Folding, in *Proceedings of the First International Meeting of Origami Science and Technology*, ed. H. Huzita (1989), 71-76.