ESTIMATING THE PLUCKING POINT ON A GUITAR STRING

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ABSTRACT

This paper presents a frequency-domain technique for estimating the plucking point on a guitar string from an acoustically recorded signal. It also includes an original method for detecting the fingering point, based on the plucking point information.

1. INTRODUCTION

Recent years have seen great advances in physical model-based synthesis. In these endeavors, knowledge of the physics and acoustics of the instruments is a theoretical starting point for the modeling. Certain simplifications can make the models computationally efficient and they can then be implemented to run in real-time on a computer. Since implemented physical models are derived from the physics of the instruments, they result in the synthesis of particularly realistic instrumental sounds. But if the physical model running on a computer is intended to be *played*, then research must be extended to the performer's action in order to understand how to interact with the computer model.

For the particular case of the classical guitar, efficient string synthesis algorithms exist and are continually being improved [1, 2, 3, 4, 5]. For the analysis counterpart, research has been undertaken in an attempt to understand the relationships between timbre nuances and model [6], physical, expressive [7, 8] and psychoacoustical [9] parameters.

Among the parameters that can be extracted, the plucking point position on the string has a major influence on the timbre nuance. The left hand fingering is crucial too. In fact, there are different ways to finger chords or play melodies. A particular fingering will be chosen because it is optimal, efficient and easy to hold, or because it sounds in a particular and desired way. Some tones on a guitar can be played with up to five different combinations of string/fret. So, if a recording is the only information available, the fingering that was used by a particular performer is not always obvious or apparent.

In this paper, a frequency-domain technique for estimating the plucking point is presented and evaluated. This paper also shows that the plucking point information can be used in order to detect the fingering on the left-hand, an important advance toward automatic score and tabulature generation.

2. PERCEPTUAL EFFECT OF THE PLUCKING POINT POSITION

Plucking a string close to the bridge produces a tone that is softer in volume, brighter and sharper. The sound is richer in high-frequency components. This happens when playing the guitar *sul ponticello*. The other extreme is playing *sul tasto*, near or over the fingerboard, closer to the midpoint of the string. In that case, the tone is louder, mellower, less rich in high frequency components. The neutral position of the right hand is just behind the sound hole. Because of the position of the right-hand fingers, the low strings are usually plucked further away from the bridge than the higher ones.

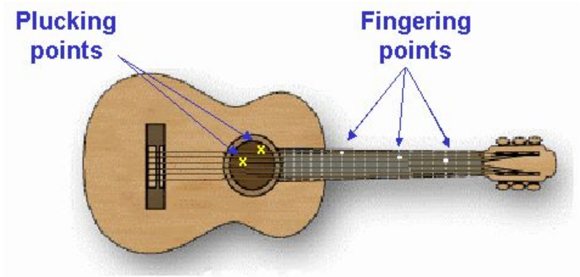

Figure 1: *Location of typical plucking and fingering points on a guitar.*

3. THEORETICAL CONSIDERATIONS

3.1. Plucking an ideal string

The plucking excitation initiates wave components traveling independently in opposite directions. The resultant motion consists of two bends, one moving clockwise and the other counterclockwise around a parallelogram. Ideally, the output from the string (force at the bridge) will lack those harmonics that have a node at the plucking point. Figure 2 illustrates a plucking position at $1/5$th of the length from one end: the spectrum will lack the harmonics that are multiples of 5.

The general solution of a vibrating string of length l with fixed ends can be written as the sum of normal modes [10]:

$$y = \sum_n (A_n \sin \omega_n t + B_n \cos \omega_n t) \sin(k_n x) \tag{1}$$

where

$$A_n = \frac{2}{\omega_n l} \int_0^l \dot{y}(x,0) \sin\left(\frac{n\pi x}{l}\right) dx, \tag{2}$$

and

$$B_n = \frac{2}{l} \int_0^l y(x,0) \sin\left(\frac{n\pi x}{l}\right) dx. \tag{3}$$

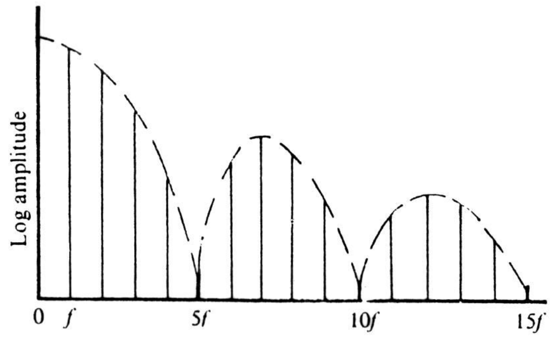

Figure 2: *Spectrum of string plucked one-fifth of the distance from one end [10].*

So, the amplitude of the nth mode is

$$C_n = \sqrt{A_n^2 + B_n^2}. \qquad (4)$$

An ideal plucking excitation at a distance p from an end and with an amplitude h is such that all points along the string have a zero initial velocity:

$$\dot{y}(x, 0) = 0 \qquad \text{for all x,} \qquad (5)$$

and the string is initially shaped like a triangle with its summit at the point (p, h):

$$\begin{aligned} y(x, 0) &= \frac{h}{p}x \qquad \text{for } 0 \leq x \leq p \qquad (6) \\ &= \frac{h(l-x)}{l-p} \qquad \text{for } p \leq x \leq l. \qquad (7) \end{aligned}$$

Therefore,

$$C_n = B_n. \qquad (8)$$

Solving the integral, it can be found that

$$C_n = \frac{2h}{n^2\pi^2 R(1-R)} \sin(n\pi R), \qquad (9)$$

where $R = p/l$ is the fraction of the string length from the point where the string was plucked to the bridge.

3.2. Plucking a real string

A real plucking differs from an ideal plucking in the following ways. The finger or plectrum exciting the string has a non-zero touching width, which adds more lowpass filtering to the excitation. A real excitation is not an event that can be modeled with linear and time-invariant operations. In fact, the finger may grab the string for a short time, while causing nonlinear or linear, but time-varying interactions. Also, the modes of the string vibration are in general nonlinearly coupled so that a mode with zero initial energy will begin to vibrate, gaining energy from other modes [11]. Finally, in the case of an acoustic guitar, the resonating body of the instrument filters the output wave of the string, according to the modes that have been excited (which depend on the plucking angle and plucking style). The forces parallel and perpendicular to the bridge excite different linear combinations of resonances, resulting in tones that have different decay rates [10].

4. ESTIMATING THE PLUCKING AND FINGERING POINT POSITIONS

In general the string is not plucked exactly at the node of any of the lowest harmonics. Since the amplitudes of the higher harmonics is considerably smaller anyway, it is not always possible to accurately detect the plucking point by simply searching for the missing harmonics in the magnitude spectrum.

The method that we investigate here for estimating the plucking point, compares the magnitude spectrum of a portion of the recorded tone to the ideal string spectra calculated for various plucking position values. Then the plucking point information is used to estimate the fingering point.

The different stages of the whole procedure are illustrated by the block-diagram in Figure 3 and are described in the following sections.

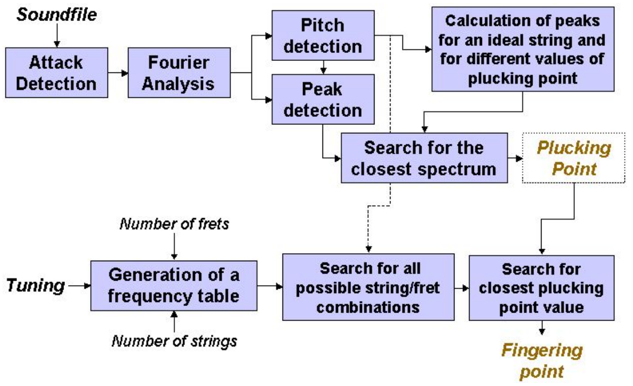

Figure 3: *Block-diagram for the estimation of the plucking and fingering points.*

4.1. Attack, pitch and peaks detection

In the first stage, the energy for successive blocks of 512 samples is calculated while an increase of the energy by a factor of 2 turns a flag on. Since it can happen that the energy increases by a factor of 2 two or more times in a row, successive alarms have to be eliminated.

After the beginning of each tone is identified, a section of the sampled waveform is chosen for analysis. The starting sample of the section is chosen at approximately $1/8$th of the distance in samples between two attacks. This roughly corresponds to the beginning of the stationary part of the sound.

The spectrum is generated by windowing the waveform and performing a long FFT (the number of bins being chosen so that two overtone peaks in the spectrum will not overlap).

2^{12} (4096) samples from the sound file are extracted, starting at the index provided by the attack detector. This length corresponds to approximately 10 periods for low frequencies. The sound portion is windowed with a Hamming window then the FFT is computed with a zero-padding factor of 6.

In order to determine the fundamental frequency, the pitch detector looks for a maximum in the spectrum provided by the Fourier analysis. Then it checks if another peak exists between 0 Hz and some frequency below the frequency of the first maximum found (a quarter of an octave below, for example). This potential peak is considered valid if it has at least 75 percent of the height of the first peak found.

This simple technique relies on the fact that the first peak (at the fundamental frequency) is generally the highest peak in the spectrum of a guitar tone. Sometimes, the highest peak is the second harmonic but the fundamental amplitude still remains significant. The above-mentioned check takes care of this special case.

Then, the harmonics (or overtones) are identified. Using the pitch value determined by the pitch detector, we look for a maximum in narrow intervals around integer multiples of the fundamental frequency.

4.2. Determination of the plucking point

The plucking point is determined from the data by finding the value of R that minimizes the absolute value of the error between the ideal string magnitude spectrum and the sampled-data spectrum, as shown in equation 10,

$$\epsilon = \sum_{n=1}^{N} \left| p_n - \left| \frac{2h}{n^2\pi^2 R(1-R)} \sin(n\pi R) \right| \right| \tag{10}$$

where p_n is the amplitude of the nth peak in the magnitude spectrum of the recorded tone excerpt.

An error surface is constructed by evaluating the error criterion for various values of R. The plucking point should correspond to minimum error, as illustrated on Figure 4.

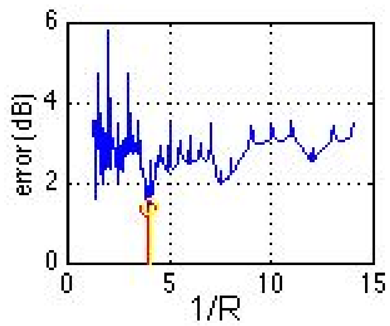

Figure 4: *Error surface for various values of $1/R$. The minimum of the error is chosen as the plucking point.*

A similar method was described in [12], although the equation for C_n contained a flaw that is corrected here in equation 9.

4.3. Determination of the fingering point

4.3.1. Ambiguity of the guitar fingering

Figure 5 illustrates the ambiguity of the fingering on a guitar. The plucking points are represented by $\times$'s and the fingering points by $\circ$'s. The bridge is the termination on the left and the nut on the right. With a standard tuning EADGBE, the same pitch would be produced in the three cases of fingering shown in Figure 5, since the finger is moved by 5 frets (corresponding to an interval of one perfect fourth) towards the nut, from the E- to the A- and the A- to the D-string. The *absolute* plucking point is the same but the *relative* plucking point is dramatically different. In fact, we have to consider the new length of the string, from the bridge to the left-hand finger pressing the string against the fret.

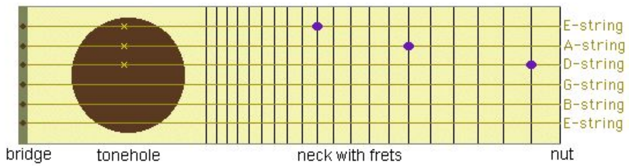

Figure 5: *Simplified representation of the neck of a guitar with plucking points ($\times$) and fingering points ($\circ$). The three string/fret combinations produce the same pitch (in this case, D-sharp).*

If we assume that the right-hand fingers pluck the strings in a narrow area close to the tonehole, the plucking point information can help to determine where the strings were fingered, by eliminating the above-mentioned ambiguity.

4.3.2. Generation of a frequency table

Given the tuning, the number of strings and the number of frets, a table of fundamental frequencies corresponding to all the string/fret intersections is generated. For a given string, the frequency is the tuning frequency if the string is open (fret 0). Since going a semitone up corresponds to a factor $2^{1/12}$, the other frequencies are obtained by multiplying the tuning frequency by $2^{F/12}$, where F is the fret index. This is illustrated in Figure 6 which displays all the pitches that can be generated on a guitar with standard tuning, as a function of the string index and of the fret index. The vertical coordinate is proportional to $\log_2$ of the fundamental frequency, in order to linearize the graph in that direction. Points at the same height correspond to the same pitch, represented by a note index. We can clearly see again the ambiguity that arises because of the fact that the same note can be played at different string/fret intersections.

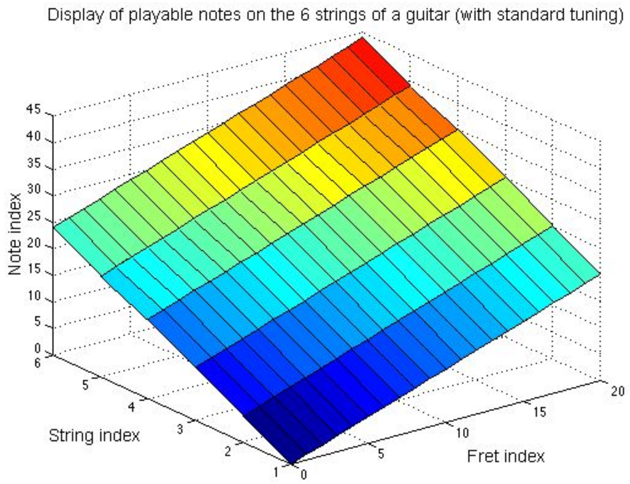

Figure 6: *Frequency table based on the tuning, the number of strings and the number of frets.*

4.3.3. Search for all possible string/fret combinations that could produce the pitch

By subtracting the detected fundamental frequency of the tone from the frequency table and taking the absolute value of the elements of the resulting matrix, a table of distances is obtained. The possible string/fret combinations are determined by searching for distances smaller than a quarter of a tone,

$$f_o \cdot \left(2^{1/24} - 1\right). \tag{11}$$

4.3.4. Search for closest plucking point value and determination of the fingering

For each string/fret combination determined at the previous stage, an approximate plucking point is calculated, assuming that the plucking is performed near the tonehole, which is at about one fourth of the strings' length:

$$\frac{D_{approx}}{l} = \frac{1}{4} \tag{12}$$

where D_{approx} is the plucking point distance from the bridge corresponding to a point in the region of the tonehole, and l is the whole length of the string.

Considering that the length of the vibrating portion of a string is shortened by a factor $2^{-1/12}$ for each semitone up[1], the approximate relative plucking point distances can be calculated for all possible string/fret combinations:

$$R_{approx}(i) = \frac{D_{approx}}{l \cdot \left(2^{-1/12}\right)^{F(i)}} \tag{13}$$

$$= \frac{D_{approx}/l}{2^{-F(i)/12}} \quad \text{for } i = 1, \dots I \tag{14}$$

where $F(i)$ the fret index of the ith possible string/fret combination.

Then the R_{approx} ratios are compared to the value for R estimated previously (based on the spectrum profile) and the closest R_{approx} will designate the fret/string combination c that is the most likely.

4.3.5. Determination of the absolute plucking point distance

Finally, knowing the fingering position and therefore the length of the vibration portion of the string and knowing the relative plucking point position (R), the absolute plucking point distance from the bridge can be determined as:

$$D = l \cdot 2^{-F(c)/12} \cdot R \tag{15}$$

5. TESTING AND RESULTS

5.1. Sound database

In order to test the algorithms, a database of recorded tones was created. Three different guitars were used:

- a hand-made 1995 Collings acoustic guitar strung with John Pearce phosphor bronze medium gauge strings
- a plywood classical guitar strung with nylon and nylon-wrapped steel Alvarez strings
- a 1953 Martin 000-18 acoustic guitar strung with John Pearce phosphor bronze light gauge strings

The tones were played with a plastic pick, .88 millimeters in thickness, triangular shaped. The intended plucking points were precisely measured and indicated on the string with a marker. The tones were recorded with a Shure KSM32 microphone in a sound-deadened room, onto digital audio tape (DAT) at 44.1 kHz, 16 bits. The microphone was placed in front of the soundhole, approximately 6 inches away, which was far enough to capture a combination of waves coming from different parts of the string, in that way limiting the filtering effect of the pick-up point.

Different series of plucks were recorded: plucks at special points along the string (1/2, 1/3, 1/4), plucks at every centimeter from the bridge to the middle of the string on open strings, chromatic scales with plucking point distance from the bridge kept constant and three-tone melodies fingered in different ways.

5.2. Results for plucking point estimation on open strings

Figure 7 displays the results for three distances from the bridge (12,13 and 14 cm).

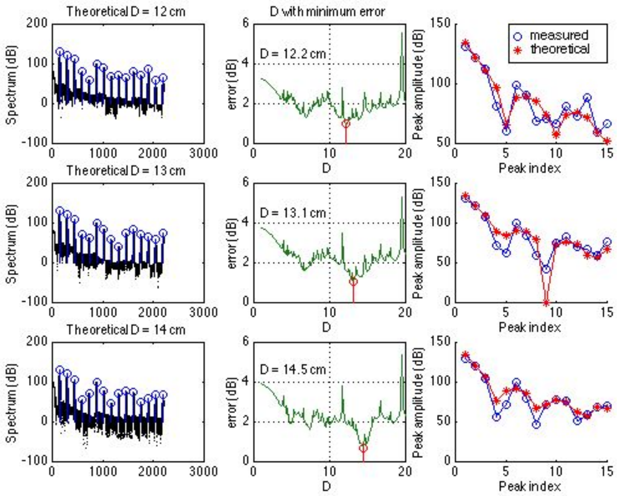

Figure 7: *Plucking point distance from the bridge D = 12, 13, 14 cm. Tones played on the classical guitar.*

The presentation of the results is as follows:

- left window: Fourier analysis of a 4096-sample portion of the sound with peak detection indicated by circles.
- middle window: error curves for various values of plucking distances D ranging from 1 to 20 cm. The minimum is indicated by a circle and the corresponding D value is displayed. The resolution of D is 0.1 cm.
- right window: comparative display of the detected peaks (o) and of the ideal string magnitude spectrum ($*$) based on the intended pluck position.

This last display was crucial in figuring out a problem of amplitude mismatch. The measured and ideal spectra have to be

[1]The string length is inversely proportional to the fundamental frequency.

matched in amplitude otherwise the computation of the error does not make sense. The code includes an autonormalization with alignment on the second harmonic. In fact, the first harmonic of the comb filter is always the highest peak, but this is not always the case for the real data spectra.

Figure 8 summarizes the results obtained for the 18 plucking points on the open A-string and open D-string. The graph displays the estimated distance versus the measured distance on the string when the tone was played. The accuracy is better than one centimeter.

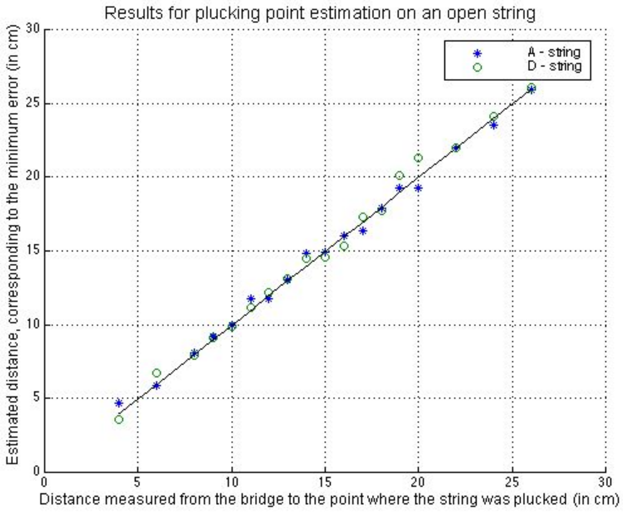

Figure 8: *Plot summarizing the results for 18 plucks on open D- and A-strings of the classical guitar.*

5.3. Results for plucking point estimation on fretted strings

Another series of tests was performed on a recording of a chromatic scale on the D-string of the classical guitar. The results for tones from D to A-sharp are summarized in Figure 9. The performance of the technique is not as good as in the case of the open string. In particular, notes A and E are the worst cases, with an error of 3.8 and 8.3 cm, respectively. This could be explained by the fact that these two notes are the most commonly used fretted notes on the D-string. Most commonly used frets wear out, making the notes “buzz” a little bit. Such a nonlinearity would tend to fill in the all-important nulls with harmonic distortion. Another possible source of extra distortion on the A and E notes is sympathetic resonance with open strings tuned at these pitches.

6. FUTURE RESEARCH

6.1. Improvements

Instead of normalizing the ideal string spectrum by scaling and shifting it, a low-order LPC filter derived from the data could be applied to the comb-filter characteristic to make it roll off like the measured spectrum.

Other types of error measures should be investigated. For example, instead of forming an error measure which has to be zero,

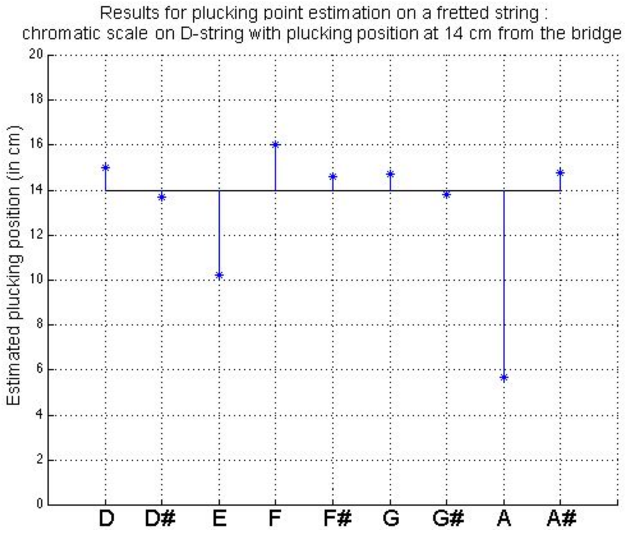

Figure 9: *Plot summarizing the results for a chromatic scale played on the D-string of the classical guitar with plucking point distance kept constant at 14 cm from the bridge.*

it is probably better to maximize the normalized inner product between the measured flattened spectral envelope (sampled at the harmonics) and the sampled harmonic comb-filter characteristic.

Also, other approaches could be taken, such as a time-domain technique using autocorrelation measures.

6.2. Application to the case of the solid-body electric guitar

The solid-body electric guitar is distinct from the acoustic guitar via two important features: (1) its body does not vibrate much, and (2) the string output is recorded by pick-up microphones placed under the strings. The pick-up systems vary across guitars. There can be up to three series of pick-up along the string, which can be put out-of-phase with one another. The timbre is greatly affected by the choice of pick-ups and the way they are combined.

In order to test the error measure on such a signal, the pick-up filtering has to be taken into account. This filtering is similar to the plucking-point effect. In fact, since we are listening to the string in one particular point, the spectrum will miss all the harmonics that have a node at that point.

If several pick-ups are used, the spectrum of the theoretical output signal can be calculated by combining the spectra at the different points along the string, using the phase information to calculate the factor that will alter each component's amplitude, in the following way:

$$\sin(\omega_n t) + \sin(\omega_n t + \phi_n) = 2\sin\left(\omega_n t + \frac{\phi_n}{2}\right)\cos\left(\frac{\phi_n}{2}\right) \tag{16}$$

where ω_n is the frequency of the nth component and ϕ_n is the phase difference due to the distance the wave has to travel from one pick-up to the next one, plus $\pi/2$ if the signals are put out of phase.

7. CONCLUSION

We presented an attempt to solve the problem of estimating the plucking point on a guitar string from an acoustically recorded signal.

The implemented technique for estimating the plucking point gives very good results for recordings of tones played on unfretted strings. The results for fretted strings are not as good, but more testing would probably help to determine the cause and a way to compensate for it.

The original method proposed for estimating the fingering point is efficient in all its stages (attack, pitch and peak detection, determination of possible fret/string intersections) but its performance depends on the performance of the plucking point estimation.

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